A. This activity will take a look at our Solar System from the perspective of scaling and proportional reasoning. I have found my students enjoy the activity and are surprised by what they see. First, a little about myself:

B. My name is Glenn Dewell. I have taught science for close to 50 years, mostly at the middle school level. I graduated from Rensselaer Polytechnic Institute in 1972 with a degree in Engineering Science. I began my teaching career at a traditional junior high in Cranston, RI and stayed there for 35 years as they transitioned to the middle school model in the mid-1990's.

C. In addition to teaching science I coordinated their enrichment program after the move to middle school. We were offered the opportunity of attending Confratute, which I declined the first summer - too busy, too much to do. But my colleagues went and when they returned they said I had to go the following summer, that Confratute was different from any professional development they had ever previously experienced. They were right, and I have now attended 16 consecutive summers.

D. I also have a passion for music, and have been a regular participant in Confratute's annual variety show. But enough about me.


1. One of the crosscutting concepts in the Next Generation Science Standards addresses scale, proportion and quantity. Proportional reasoning involves thinking about relationships and making comparisons of quantities or values. In the words of John Van de Walle, "Proportional reasoning is difficult to define. It is not something that you either can or cannot do but is developed over time through reasoning ... It is the ability to think about and compare multiplicative relationships between quantities". Scale and proportion are often best understood using models. For example, the relative scales of objects in the solar system or of the components of an atom are difficult to comprehend mathematically (because the numbers involved are either so large or so small), but visual or conceptual models make them much more understandable (e.g., if the solar system were the size of a penny, the Milky Way galaxy would be the size of Texas).

2. The following activity approaches these concepts by comparing the planets in the solar system, using both size and distance from the Sun. For middle school the only materials needed are a sheet of 11 " by 17 " paper, a metric ruler, and a pencil. But I will address how to scale it up to a larger size and present ideas for using this at the elementary level. I include Pluto to represent dwarf planets, and because I received a gift from a student last year - the bookmark shown here.

3. The activity also lends itself to decreasing the apprehension students can feel about working with fractions and I present the ideas behind scaling as a simple comparison and show that in fractional form. Another important skill that is addressed is estimation, and I will suggest some hints on how to practice that with students.
Estimation

## The fine art of guessing


6. Depending on the group, my first step is to ask the students how we can compare the distances of planets to the Sun. We talk about how a scale is created, often using as the base unit something that is familiar to us (a king's foot, for example). You can go on a tangent about this to discuss "standard" units and why using a king's foot can create problems for merchants and customers alike.

7. I then ask my students to make their best guess as to the distances to the Sun of each planet and see how close they can come to the actual figures. (No looking up the answers!) We end up with a chart that looks like this:

8. Now I introduce the concept of scale, and why it is necessary for an accurate model of the solar system. I ask my students to propose the "standard" and lead them to the idea of using our home, the Earth, as our base unit. We end with our base unit being the average distance from the Earth to the Sun, or one "Earth". Astronomers have a name for this unit: an astronomical unit. (Once again, there are many tangential discussions that can veer off for a while depending on the ages of your students and what you have covered previously. The idea that orbits are ellipses, not perfect circles. How Kepler came to embrace this idea despite the beliefs to the contrary during his time. And, once they understand that
distance to the Sun varies, try asking them to name the month when the Earth is closest to the Sun (the answer is January). Some students usually know this, but you can have an interesting debate as students try to support and rebut their claims.

## Here is a map showing 3 towns. <br> On the map 1 cm represents 3 miles.


9. Without using any calculators, I ask students to estimate how many "Earth" distances each planet is from the Sun. We try to arrive at an acceptable consensus for each. They can use the chart we created, but I strictly enforce the "no calculator" rule for this part.

10. We then discuss the idea of a fraction as a comparison between two numbers. Using some non-astronomical examples I relate fractions to decimals and ratios saying repeatedly "It's just comparing one number to another". Some fractions I might use are $3 / 4,7 / 8,3 / 2$ and $8 / 3$, depending on the age group. My main goal is to compare: Is $3 / 4$ more or less than $1 / 2$ ? Is $7 / 8$ closer to 1 or $1 / 2$. Is $3 / 2$ bigger or smaller than 1 ?

11. To check our estimates we use the actual distances to create fractions that put the distance of each planet to the Sun into "Earth" units. Using calculators we convert the fractions to decimals, reminding students we are just comparing two numbers. I usually round off to one place after the decimal point. If you are working with younger students you can, with the exceptions of Mercury and Venus, round off to the nearest whole number. Some examples are:

Venus: $\quad 108$ million km_ $\quad=0.72$ Earths $=0.7$ Earths 150 million km

Uranus: _2,871 million km_ = 19.14 Earths = 19.1 Earths 150 million $\mathbf{k m}$

NOTE: You can show dimensional analysis here also
11A. Our final chart will look like this:

| Planet | Distance to Sun (kilometers) | $\begin{aligned} & \text { Distance to Sun } \\ & \text { (Earths) } \end{aligned}$ |
| :---: | :---: | :---: |
| Mercury | 58,000,000 km | 0.4 |
| Venus | 108,000,000 km | 0.7 |
| Earth | 150,000,000 km | 1.0 |
| Mars | 228,000,000 km | 1.5 |
| Jupiter | 778,000,000 km | 5.2 |
| Saturn | 1,427,000,000 km | 9.5 |
| Uranus | 2,871,000,000 km | 19,1 |
| Neptune | 4,497,000,000 km | 30 |
| *Pluto | 5,913,000,000 km | $39.4$ |

12. I then tell the students we are going to make a model of the Solar System distances on a 11 " by 17 " paper. That means making one more scale: One "Earth" will equal one centimeter. You can show students why this unit makes sense by measuring the length of the paper, about 43 centimeters, and discussing what scale would allow them to fit the maximum distance (the 39.4 Earths to Pluto) on the paper. There are connections about how to scale the axes of a graph here.

17 inches $=43.18$ centimeters.
Need to fit 39.4 Earths
Dividing 43.18 centimeters by 39.4 Earths tells me that each "Earth" can be NO BIGGER THAN 1.095939 centimeters (dimensional analysis). When we set up any scale (the axis of a graph, for instance), we look for next SMALLER, EASIER number to use. In this case 1.0 is the best choice.
13. Now the fun part: With the 11 " by 17 " paper in landscape orientation, have the students put a small (very) dot on the far left or right of the paper and label it the Sun. For younger students I mark off the line down the middle of the paper in centimeters. For more advanced students I simply give them a metric ruler and let them figure it out. They are then challenged to put a small dot the appropriate distance from the Sun for each planet, and to label each planet. (A common mistake is for students to measure from one planet to the next.) The fact that Earth is only one centimeter from the Sun, with Mercury and Venus even closer, is an eye opener. As is the fact that the spaces between the other planets are much greater. Here's how a final drawing might appear.

15. On the opposite side of the paper we create a new scale based on the sizes of the planets. The consensus will be to use diameters, and I will tell them the Earth's diameter is estimated as 12,756 kilometers. You can round these numbers off for younger students. We repeat the process of having students make their best guess as to the diameters of the other planets and see how close they can come to the actual figures. We end up with a chart that looks like this:

| Planet | Diameter (kilometers) | Diameter (Earths) |
| :---: | :---: | :---: |
| Mercury | 4.878 km |  |
| Venus | 12,104 km |  |
| Earth | 12,756 km |  |
| Mars | 6,794 km |  |
| Jupiter | 142,800 km |  |
| Saturn | $120,540 \mathrm{~km}$ |  |
| Uranus | 51,200 km |  |
| Neptune | 49,500 km |  |
| *Pluto | 2,200 km |  |

16. We return to the idea of using our home, the Earth, as our base unit. Without using any calculators, I ask students to estimate how many "Earths" there are in each of the planets. We try to arrive at an acceptable consensus for each. I strictly enforce the "no calculator" rule for this part. Then, as we did with distances to the Sun, we create fractions that compare the diameter of each planet to the Earth.

Mercury __ $4,878 \mathrm{~km}$
$12,756 \mathrm{~km}$

| Mars | $\frac{6,794 \mathrm{~km}-}{12,756 \mathrm{~km}}$ |
| :--- | :--- |
| Jupiter | $-\frac{142,800 \mathrm{~km}}{12,756 \mathrm{~km}}$ |

Neptune $\quad-\frac{49,500 \mathrm{~km}}{12,756 \mathrm{~km}}$
18. We again use calculators to compare the diameter of each planet to the Earth's diameter. I reinforce how to "round off" numbers. Once again, you can adjust this for the age group.
Mercury $\underset{12,756 \mathrm{~km}}{\underline{4,878} \mathrm{~km}} \quad=0.3824082$ "Earths" $=0.4$ "Earths"
Mars $\frac{6,794 \mathrm{~km}}{12,756 \mathrm{~km}} \quad=0.532612$ "Earths" $=0.5$ "Earths"

Jupiter $\quad-\frac{142,800 \mathrm{~km}}{12,756 \mathrm{~km}} \quad=11.194731$ "Earths" $=11.2$ "Earths"
Neptune $-\frac{49,500 \mathrm{~km}}{12,756 \mathrm{~km}} \quad=3.8805268$ "Earths" $=3.9$ "Earths"

Here's an example of our final chart.

19. At this point I tell them to use the same concept for a scale as on the other side: 1 "Earth" diameter equals 1 centimeter. I only give my students a metric ruler as a tool and challenge them to figure out how to create a reasonable circle without a compass or other tool. I also challenge them to add color and other enhancements to their project. A final project might look like this.

20. NOTE: If a student asks if they can put the Sun on the paper, I challenge them to explain why or why not. At the scale we are using, one Earth diameter equal to one centimeter, the Sun's diameter would equal 109 centimeters. Here's a picture to show that to scale.

21. You can scale this activity up to whatever you want. For example, if you have a baseball field with at least 200 feet from home plate to outfield fence, you could use a scale where the Earth is 5 feet from the Sun and Pluto's average distance would be just under 200 feet from the Sun. I will note that Pluto's orbit is very elliptical and its greatest distance from the Sun is almost 50 astronomical units (the average distance of the Earth from the Sun). Pluto is actually closer to the Sun for about 20 years out of its 240 year orbital period. That last happened between 1979 and 1999, a trick question I would use on my test on the Solar System.
22. No matter how big a field you have, it is not practical to use the same scale for size of the planets and their distances to the Sun. If the Earth were the size of a marble (about 2 cm ), the Sun would be 2.5 times the size of a typical yoga ball on which you sit. Mars would be the size of a pea, Venus a marble like the Earth, Jupiter a pro soccer ball, Saturn a gym-class handball, with Uranus and Neptune the size of softballs. Mercury and Pluto would be smaller than the pea used for Mars. Earth would be 235 meters away from the Sun. Saturn would be 2,237 meters - about 1.4 miles - away. Neptune would be just over 7 kilometers (almost 4.4 miles) away. I've attached a link for just such a model, created by an enterprising group.

NGSS Scale, Proportion and Quantity

## Scale, Proportion and Quantity

Scale, Proportion and Quantity are important in both science and engineering. These are fundamental assessments of dimension that form the foundation of observations about nature. Before an analysis of function or process can be made (the how or why), it is necessary to identify the what. These concepts are the starting point for scientific understanding, whether it is of a total system or its individual components. Any student who has ever played the game "twenty questions" understands this inherently, asking questions such as, "Is it bigger than a bread box?" in order to first determine the object's size.
An understanding of scale involves not only understanding systems and processes vary in size, time span, and energy, but also different mechanisms operate at different scales. In engineering, "no structure could be conceived, much less constructed, without the engineer's precise sense of scale... At a basic level, in order to identify something as bigger or smaller than something else—and how much bigger or smaller—a student must appreciate the units used to measure it and develop a feel for quantity." (p.90) "The ideas of ratio and proportionality as used in science can extend and challenge students' mathematical understanding of these concepts. To appreciate the relative magnitude of some properties or processes, it may be necessary to grasp the relationships among different types of quantitiesfor example, speed as the ratio of distance traveled to time taken, density as a ratio of mass to volume. This use of ratio is quite different than a ratio of numbers describing fractions of a pie. Recognition of such relationships among different quantities is a key step in forming mathematical models that interpret scientific data." (p. 90)
The crosscutting concept of Scale, Proportion, and Quantity figures prominently in the practices of "Using Mathematics and Computational Thinking" and in "Analyzing and Interpreting Data." This concept addresses taking measurements of structures and phenomena, and these fundamental observations are usually obtained, analyzed, and interpreted quantitatively. This crosscutting concept also figures prominently in the practice of "Developing and Using Models." Scale and proportion are often best understood using models. For example, the relative scales of objects in the solar system or of the components of an atom are difficult to comprehend mathematically (because the numbers involved are either so large or so small), but visual or conceptual models make them much more understandable (e.g., if the solar system were the size of a penny, the Milky Way galaxy would be the size of Texas).

Proportional reasoning involves thinking about relationships and making comparisons of quantities or values. In the words of John Van de Walle, "Proportional reasoning is difficult to define. It is not something that you either can or can not do but is developed over time through reasoning ... It is the ability to think about and compare multiplicative relationships between quantities"

