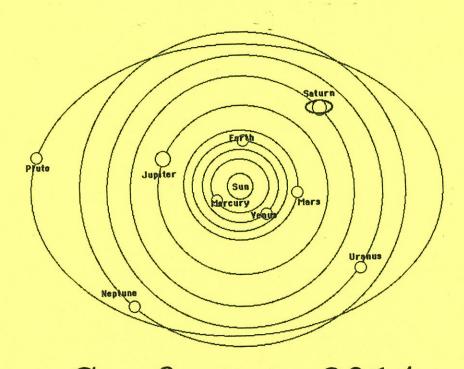
Astronomy For Kids Of All Ages



Confratute 2014
Glenn Dewell
gdewell@hotmail.com

This session is conducted by Glenn Dewell (long time physics and general science teacher in Rhode Island) and Dr. Dennis Machnik (physics and astronomy professor and Director of the Mark Sylvestre Planetarium at Plymouth State University). It will address a variety of topics in astronomy and help to correct misconceptions that teachers may have about the subject. Some of the ideas we hope to touch on are:

- *Observing that the sun, moon and stars appear to move slowly across the sky.
- *Observing that the moon looks slightly different from day to day, but looks the same again in about four weeks.
- *Observing that the sun can only be seen in daytime, but the moon can be seen sometimes at night a sometimes during the day.
- *Using models to describe the relative motion/position of the Earth, sun and moon.
- *Defining the Earth's gravity as a force that pulls any object on or near the Earth toward its center without touching it.
- *Recognizing that the sun is the center of our solar system, the Earth is one of several planets that orbit the sun, and that our Moon orbits the Earth.
- *Recognizing that it takes approximately 365 days for the Earth to orbit the sun.
- *Identifying and comparing the size, location, distances, and movement (e.g. orbit of planets, path of meteors) of the objects in our solar system.
- *Observing that there are more stars in the sky than can easily be counted, but they are not scattered evenly and not all the same brightness.
- *Recognizing that throughout history people have identified patterns of stars that we call constellations.
- *Describing the apparent motion/position of the objects in the sky (e.g constellations, planets).
- *Identifying the sun as a medium-sized star located near the edge of a disk-shaped galaxy of stars.

Ancient Greek physics

keen observations, no basic laws
no predictions
no correlation
buoyancy, atmosphere
atomic theory
math important
geometry

Pythagoras (560-480 B.C.E.) numbers everything symmetries, harmonies famous right triangle theorem

Aristotle (384-322 B.C.E.)

tutor of Alexander the Great
student of Plato
founded The Lyceum
classification system based on math
two kinds of motion
natural motion
Earth – straight up or down
normal state –rest, proper place
Celestial – circular
perfect spheres made of "ether"
Moon corrupted by Earth

violent motion
no motion without force
continual mover
no concept of inertia

Euclid (325-270 B.C.E.) father of geometry (non-Euclidian geometry-elliptical, hyperbolic)

Aristarchus (310-230-B.C.E.)
heliocentric idea
measured distance to sun, moon
sun bigger than Moon, Earth
Moon, Earth go around Sun

Archimedes (287-212 B.C.E.)

buoyancy principle

found Pi geometrically

difference between finite, infinite

developed ingenious defensive devices

Erastosthenes-(276-194 B.C.E.) measured Earth's circumference

Hipparchus (190-120 B.C.E)
greatest of ancient astronomers
discovered precession
stellar magnitudes
invented trigonometry

Ptolemy (100-170 C.E.).
summarized Greek astronomy (Almagest)
geocentric universe
epicycles - planetary motion

From Ptolemy to Newton

Aristotle's ideas adopted by Church
geocentric universe
little science during Dark Ages
technology did develop
navigation, clocks, gunpowder, paper making
printing press (1436)
Leonardo DaVinci

Nicolaus Copernicus (1473-1543)

heliocentric theory

circular orbits - theological reasons

dangerous

Tycho Brahe (1546-1601)
very accurate naked eye observations
instruments to map sky
King of Denmark - patron

Johannes Kepler (1571-1630)

Brahe's assistant

3 laws of planetary motion
ellipse - shape of orbits
data used to judge theories

Laws of planetary motion:

- 1. all planets move in an ellipse around the Sun Sun at one of two focal points discovered empirically (trial and error) no uniform speed for planets
- 2. line from Sun to planet sweeps out equal areas in equal times
- 3. the square of a planet's period of revolution is proportional to the cube of its mean distance

Galileo Galilei (1564-1642)

demonstrated shortcoming of Aristotelian physics continuous motion did not required constant pushobjects fall at same rate with no air resistance

advanced experiments and observations not. philosophical speculation

established kinematics as a science identified inertia

studied freely falling bodies importance of air resistance used inclined planes

studied projectile motion horizontal, vertical components 45 degree angle give maximum distance-

used pendulums to measure time

invented the thermometer

improved (did not invent) telescope
observed Moon
observed four of Jupiter's moons, phases of Venus
Milky Way - made of very distant starts
heliocentric universe

his ideas were dangerous challenged Earth's central place, uniqueness

Things a Second Grader should know about Astronomy

- 1. The sun can only be seen in the daytime, but the moon can be seen sometimes at night and sometimes during the day.
- 2. The sun, moon and stars appear to move slowly across the sky.
- 3. The moon looks slightly different from day to day.
- 4. There are more stars in the sky than can easily be counted, but they are not scattered evenly and not all the same in brightness.

ASTRONOMY STUFF A FOURTH GRADER SHOULD KNOW

- 1. The sun can only be seen in the daytime, but the moon can be seen sometimes at night and sometimes during the day.
- 2. The sun, moon and stars appear to move slowly across the sky.
- 3. The moon looks slightly different from day to day, but looks the same again in about 4 weeks.
- 4. The rotation of the Earth on its axis every 24 hours produces the day/night cycle.
- 5. The sun is the center of our solar system and Earth is one of several planets that orbit the sun, and the moon orbits the Earth.
- 6. It takes about 365 days for the Earth to orbit the sun.
- 7. There are more stars in the sky than can easily be counted, but they are not scattered evenly and not all the same in brightness.
- 8. Throughout history people have identified patterns of stars that we call constellations.

Astronomy Tasks for Eighth Graders

- 1. Identify and compare the size, location, distances, and movement (e.g. orbit of planets, path of meteors) of the objects in our solar system.
- 2. Compare the composition, atmosphere, and surface features of objects in our solar system.
- 3. Identify major discoveries from different scientists and cultures and describing how these discoveries have contributed to our understanding of the solar system (e.g. timeline, research project, picture book).
- 4. Use models to describe the relative motion/position of the Earth, sun and moon.
- 5. Explain night/day, seasons, year, and tides as a result of the regular and predictable motion of the Earth, sun, and moon.
- 6. Define the Earth's gravity as a force that pulls any object on or near the Earth toward its center without touching it.
- 7. Describe the relationship between mass and the gravitational force between objects.
- 8. Describe the relationship between distance and the gravitational force between objects.
- 9. Explain that the sun's gravitational pull holds the Earth and other planets in their orbits, just as the planet's gravitational pull keeps their moons in orbit
- 10. Use or create a model of the Earth, sun and moon system to show rotation and revolution
- 11. Use a model of the Earth, sun and moon to recreate the phases of the moon.
- 12. Describe the apparent motion/position of the objects in the sky. (e.g. constellations, planets).
- 13. Identify the sun as a medium-sized star located near the edge of a disk-shaped galaxy of stars.
- 14. Describing the universe as containing many billions of galaxies, and each galaxy contains many billions of stars.

High School Astronomy GSEs

Explain the role of stars in navigation, beginning with ancient civilizations, advancing through 19th century mathematical celestial navigation, to current Global Positioning Systems.

Explain how scientific knowledge regarding the structure of the universe has changed over time due to advances in technology which accumulates new evidence to redefine scientific theories and ideas.

Compare the processes involved in the life cycle of stars (e.g. gravitational collapse, thermonuclear fusion, nova) and evaluate supporting evidence.

Explain how the "Big Bang" theory has developed over time citing evidence to support its occurrence (Doppler Effect/red shift).

Applying the properties of waves/particles to explain the movement, location, and composition of the stars and other bodies in the universe.

Relating the process of star formation to the size of the star and including the interaction of the force of gravity, fusion, and energy release in the development of the star

Identifying and describing the characteristics common to most stars in the universe.

Describing the ongoing processes involved in star formation, their life cycles and their destruction.

MOON FACTS

- 1. Moon may have formed when a Mars-sized object struck the early Earth a. Earth got most of the iron
- 2. Moon is ancient, preserves an early history
- 3. Moon once covered by molten lava
 - a. Perhaps melted completely
- 4. Darker maria regions and lighter highland regions have different kinds of rocks
- 5. Highland regions Moon's mountains found here
 - a. remains of earliest crust of Moon
- 6. Maria regions dark lava flows where early crust was punctured
 - a. Far side of Moon has very few maria regions
- 7. Moon is lifeless
- 8. No atmosphere
 - a. Stars visible during lunar daytime
 - b. No protection from harmful solar radiation
- 9. No water, except in some deep moon craters near the lunar poles.
- 10. No magnetic field star maps used to find way around
- 11. Preserves effects of intense meteorite bombardment
 - a. Weathering and erosion on Earth has wiped out effects here
- 12. Moon divided into outer crust, inner mantle, possible small metal core
- 13. Slightly egg-shaped
- 14. Surface covered by powdery layer of dust

Other Moon facts:

- 15. Long daylight (two Earth weeks) followed by equally long night
- 16. Over 100 degrees Celsius in middle of lunar day
- 17. Over 150 degrees BELOW ZERO (Celsius) during long lunar night
- 18. Large craters overlap
 - a. some craters 250 km (over 150 miles) across
- 19. Only one side of the Moon faces the Earth
 - a. all sides of Moon have a day and night period
- 20. Moon's diameter is about 1/4 that of Earth's
- 21. Moon's gravity is about 1/6 that of Earth's
 - a. impact materials thrown farther
 - b. easy movement

LOST ON THE MOON

You are one of a team of ten astronauts on a mission in the highlands region of Earth's moon. The United States has been exploring the moon in preparation for the building of a permanent base there.

You and the other members of your group (3 to 5 of the astronauts) are returning to the base ship after exploring possible sites for bauxite and magnetite mines when your vehicle crashes. The accident occurs in the middle of includes daylight period, about 150 kilometers from the base ship.

Your survival depends on reaching the base ship. Most of your equipment has been destroyed. In addition to your space suits, you are able to remove the following items from the wreckage:

50 meters of nylon ROPE
portable, battery-powered HEATER
3 tanks of OXYGEN (60 hours each)
MATCHES (6 boxes)
STAR MAP
magnetic COMPASS
WATER (6 ten liter tanks)
SIGNAL FLARES (chemically activated)
TWO-WAY RADIO (40 kilometer range)
FOOD concentrate, one case
FIRST AID KIT
dehydrated MILK (one case)
battery powered FLASHLIGHT
nylon FABRIC (4 meters by 4 meters)

Rank each item on the list according to how important it will be for your survival as you try to return to the base ship. The most important item should be ranked first and the least important item last. Any extra time that you have should be used to tell why you ranked each item where you did.

NAMES	· ·			<u> </u>	 	<u>-</u>				
RANK	ITEM		USE	S						
1.		*								-
2.		*			, , , , , , , , , , , , , , , , , , , ,					
3.		*								·
4.		*								
5.		*			 				 :	<u> </u>
6.		*								
7.		*								-
8.		*								
9.		*								
10.	. <u></u>	*								-
11.		*							·	•
12.		*							···	
13.		*					-	-		
14.		*		<u> </u>						 -

Lost on the Moon Scoring

POSITION -D	1	2	ſ 3	4	r 5	6	7	8	9	10	11-14	j,
Oxygen	5	3	0	6	0	6	0	0	0	0	0	-
<u>Water</u>	3	5	2	0	6	0	0	0	0	0	6	+
<u>Food</u>	0	3	5	5	3	2		l	o	0	6	
Star Map	0	2	5	5	3	2	2	2	0	0	0	†
Rope	0	0	0	2	5	5	5	4	2	1	6	j:
Nylon Fabric	O	0	0	2	5	5	5	4	2	l	Ö	-
<u>Radio</u>	0	0	0	2	4	5	5	5	2	1	0	!
First Aid Kit	0	0	٥	0	2	コ	3	5	5	5	2	
Dehydrated Milk	0	0	0	0	l	1	2	3	5	5	2	- •
Flares	0	0	0	D	0	0	2	3	5	S	2	•
<u>Heater</u>	0	0	G	0	0	0	0	0	D	#	5	-
<u>Flashlight</u>	0	0	0	0	0	0	0	0	0	1	5	<u>.</u>
Magnetic Compass	0	6	0	0	0	O	0	0	0	1	5	•
<u>Matches</u>	0	0	0	0	0	0	0	0	0	1	S	- -

SUPER GIANTS. GIANTS, WHITE DWARFS

SUPER GIANTS

Betelgeuse (3100K, -6) Antares (3500K, -5) Rigel (11,000K, -7) Deneb (9,000K, -7)

GIANTS

Arcturus (4500K, 0) Aldebaran (4000K, -1) Capella (6000K, -1) Polaris (6600K, 0)

WHITE DWARFS

Procyon B (8000K, +11) Sirius B (25000K, +8) Van Maanen's Star (6500K, +12)

MAIN SEQUENCE STARS

TYPE B

Bellatrix (21,500K, -3) Spica (25,000K, -4) Regulus (15,000K, -1) Algol A (12,500K, 0)

TYPE A

Sirius A (9400K, +1) Mizar (9600K, 0) Alcor (8500K, +2 Altair (7500K, +2) Zubenelgenubi (8500K, +1) Vega (10,000K, 0)

TYPE F

Procyon A (6600K, +3) Eta Cassiopeia (6000K, +3) Pi Orionis (6400K, +4)

TYPE G

Alpha Centauri (5700K, +4) Sun (5800K, +5) Chi Tauri (6000K, +3) Delta Trianguli (6000K, +4)

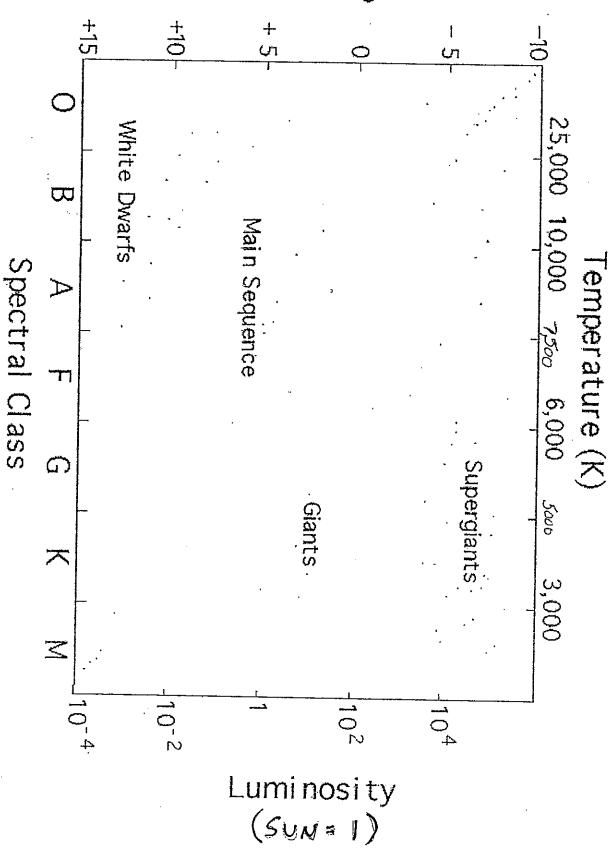
TYPE K

Pollux (4500K, +8 Algol B (4500K, +9 Epsilon Eridani (5000K, +6

TYPE M

Proxima Centauri (3000K, +15) Barnard's Star (3100K, +13) Kapteyn's Star (3500K, +11

Absolute Magnitude



Newton's Laws of Motion

The English physicist Sir Isaac Newton is most widely known for his three laws of motion. Here is a brief description of each.

NEWTON'S FIRST LAW OF MOTION: Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed upon it.

Often called the Law of Inertia, the key word is *continues*. If it is not moving, a body will continue to not move unless a net force is acting on it. If a body is moving, it will continue to move without changing its speed or turning. When we measure the mass of an object, we are measuring its inertia, or resistance to change in motion.

NEWTON'S SECOND LAW OF MOTION: The acceleration of a body is directly proportional to the net force acting on the body and inversely proportional to the mass of the body and is in the direction of the net force.

Often expressed as FORCE equals MASS times ACCELERATION, or F = MC

Newton really said
$$Q = \frac{F}{m}$$

For a given mass, if you double the force you double the acceleration. If you triple the force, you triple the acceleration. For a given force, if you double the mass you get half the acceleration. If you triple the mass, you get one third the acceleration.

NEWTON'S THIRD LAW OF MOTION: Whenever a body exerts a force on a second body, the second body exerts an equal and opposite force on the first body.

This is often stated this way: "For every action there is an equal and opposite reaction." It is important to understand that the action and reaction forces act on different bodies, and don't cancel each other out. They make up a pair of forces. Since neither exists without the other, it doesn't matter which force is the action force and which is the reaction force.

Physics Lingo

An amusement park is a great place to think about some simple physics concepts. At an amusement park, your mind and body are exposed to many out of the ordinary stimuli. This is what supplies much of the "amusement" at the park. Instead of the slow up and down of an elevator, you will be close to free fall! The small centripetal forces you feel going around a corner in a car will be replaced by forces strong enough to pin you to the ride's walls! Since your body is not used to such extended and extreme stimuli, it will provide you with some unconditioned responses. To accurately describe your experiences, you must be able to speak the language of science. Here are a few key terms.

ACCELERATION - rate of change in speed and/or change in direction of motion.

AIR RESISTANCE - force of the air pushing against a moving object.

- CENTER OF MASS The average position of all the particles of mass in a body. It is the same point as the center of gravity, which is the gravitational "middle" of an object and the average position of its weight.
- CENTRIPETAL FORCE The force that pulls or pushes an object toward the center of its circular path. Not to be confused with centrifugal force, a false force that seems to be pushing an object away from its circular path.

FORCE - A push or a pull

FRICTION - Resistance to motion between two surfaces in contact.

GRAVITY - Force of attraction between all bodies in the universe.

- G-FORCE Comparison of the force felt to the force that an object would feel while motionless on the earth's surface a force of 1g represents normal gravity greater than 1g, you feel "heavier", less than 1g you feel "lighter".
- INERTIA Tendency of matter to remain at rest or to keep moving in a straight line.

 When you measure the mass of an object, you are measuring its inertia.
- KINETIC ENERGY Energy that an object has due to its motion.
- MASS The amount of matter in an object when we measure mass, we are measuring the amount of inertia an object has.
- POTENTIAL ENERGY Energy an object has due to its position objects that can fall have gravitational potential energy.

WEIGHT - The measure of the gravity pull between an object and the earth

WEIGHTLESSNESS - A condition in which an object feels no net forces

ROLLER COASTERS

Last but not least, here's some roller coaster stuff. True coasters pull you to the top of the first hill, storing GRAVITATIONAL POTENTIAL ENERGY (energy due to an object's position as a result of Earth's gravity). This energy will carry the passengers from start to finish. To think about the energy involved you need to know a little about THERMODYNAMICS and CONSERVATION OF ENERGY.

Thermodynamics is the study of heat and its transformation to mechanical energy. The word "thermodynamics" comes from the Greek words meaning "movement of heat". Its foundation is the conservation of energy and the fact that heat flows from hot to cold and not the other way around.

The LAW OF CONSERVATION OF ENERGY can be stated as follows: "Energy cannot be created or destroyed; it may be transformed from one form into another, but the total amount of energy never changes."

On most coasters, a gasoline engine is used to generate mechanical energy to pull the ride to the top of the first hill. This energy is converted into KINETIC ENERGY (the energy of motion) as you roll down the first hil. It is this kinetic energy that will carry you to the end of the ride.

Think about where the potential energy can from and trace it back as far as possible. Think about where the kinetic energy you gained rolling down the first hill goes. Remember, it all had to go somewhere. Also, explain why the first hill on a true coaster is always the highest.

If you have a chance to ride a coaster, try to observe places where you feel especially light or especially heavy.

CLOTHOID LOOP: The days of simple roller coasters with a few small hills are long gone. Today's coasters are bigger and scarier than ever, many having loops. Early-day looping coasters had circular loops. At the top of the loop the carts began to slow down, and at the end of the loop they were going too fast. The development of the clothoid loop solved the problem. By decreasing the loop's radius at the top, the speed is increased enough to keep passengers pressed into their seats. At the same time, the radius at the bottom of the loop is increased so that the curve isn't sharp enough to hurt people. The result is a loop that is more elliptical than circular.

RADIANT ENERGY: ENERGY TRANSFERRED FROM ONE PLACE TO ANOTHER BY WAVES.

EXAMPLES ARE LIGHT, HEAT (INFRARED), X-RAYS, GAMMA RAYS AND RADIO WAVES.

WAVELENGTH: HORIZONTAL DISTANCE BETWEEN WAVES.

FREQUENCY: NUMBER OF WAVES THAT PASS BY A POINT EACH SECOND.

ELECTROMAGNETIC SPECTRUM: FORMS OF RADIANT ENERGY ARRANGED FROM VERY LONG WAVELENGTHS (RADIO WAVES) TO VERY SHORT WAVELENGTHS (GAMMA RAYS).

*VISIBLE LIGHT IS ONLY A VERY SMALL PART OF THE E-M SPECTRUM.

- * THE VISIBLE LIGHT SPECTRUM RANGES FROM RED (LONG) TO VIOLET (SHORT).
- * OUR ATMOSPHERE BLOCKS MOST ELECTROMAGNETIC RADIATION. MAINLY, VISIBLE LIGHT AND RADIO WAVES PENETRATE TO EARTH'S SURFACE.

ASTRONOMICAL UNIT: AVERAGE DISTANCE FROM EARTH TO SUN. USED TO MEASURE DISTANCES WITHIN THE SOLAR SYSTEM.

LIGHT YEAR: DISTANCE THAT LIGHT TRAVELS IN ONE YEAR (ABOUT 6 TRILLION MILES).
USED TO MEASURE DISTANCES BETWEEN STARS.

PARSEC: ALSO USED TO MEASURE VERY LARGE DISTANCES. EQUAL TO 3.26 LIGHT YEARS.

NICHOLAS COPERNICUS: POLISH SCIENTIST WHO SUGGESTED THAT THE SUN (NOT THE EARTH) WAS THE CENTER OF THE SOLAR SYSTEM. GALILEO WAS JAILED FOR DEFEND-ING THIS IDEA.

TYCHO BRAHE: DANISH MATHEMATICIAN WHO MADE 20 YEARS OF ACCURATE RECORDINGS OF THE POSITIONS OF STARS AND PLANETS (WITHOUT THE USE OF A TELESCOPE).

JOHANNES KEPLER: GERMAN ASTRONOMER AND MATHEMATICIAN AND ASSISTANT TO TYCHO BRAHE. PROPOSED MATHEMATICAL DESCRIPTIONS OF PLANETS' ORBITS AROUND THE SUN.

GALILEO GALILEI: ITALIAN ASTRONOMER AND PHYSICIST. IMPROVED THE REFRACT-ING TELESCOPE AND DISCOVERED IMPORTANT NEW FACTS ABOUT ASTRONOMY.

SIR ISAAC NEWTON: ENGLISH SCIENTIST WHO INVENTED THE REFLECTING TELESCOPE.

WORK IN OPTICS LAID FOUNDATION FOR SPECTRAL ANALYSIS. MOST FAMOUS FOR
HIS THREE LAWS OF MOTION.

ALBERT EINSTEIM: REVOLUTIONIZED SCIENTIFIC THOUGHT WITH NEW IDEAS ABOUT TIME, SPACE, MASS, MOTION AND GRAVITY. MOST FAMOUS FOR HIS THEORY OF: RELATIVITY.

OPTICAL TELESCOPES (COLLECT VISIBLE LIGHT)

- 1. REFRACTING TELESCOPE- USES A LENS TO GATHER AND FOCUS LIGHT
- 2. REFLECTING TELESCOPE: USES A MIRROR TO GATHER AND FOCUS LIGHT. CAN BE MADE MUCH LARGER THAN A REFRACTING TELESCOPE.

RADIO TELESCOPES

COLLECT AND FOCUS RADIO WAVES INSTEAD OF LIGHT CAN BE USED DAY AND NIGHT AND IN ANY WEATHER CAN PROBE DEEPER INTO SPACE

**SEVERAL RADIO TELESCOPES GAN BE LINKED BY COMPUTERS TO ACT AS A SINGLE, VERY LARGE TELESCOPE: THIS IS CALLED A VLA (VERY LARGE ARRAY).

SPECTROSCOPES

BREAKS VISIBLE LIGHT INTO ITS SPECTRUM

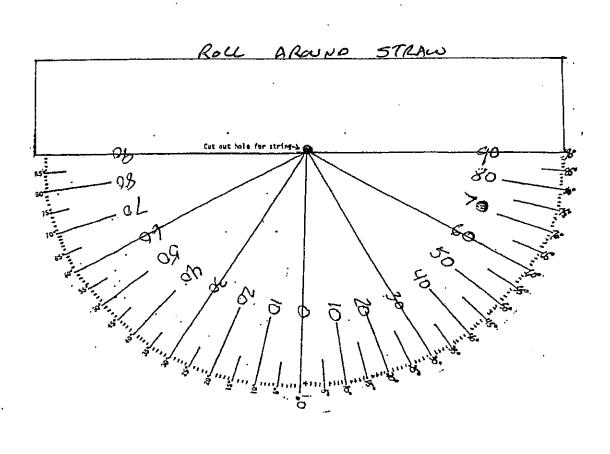
EACH ELEMENT EMITS ENERGY WITH WAVELENGTHS THAT ARE LIKE FINGERPRINTS,

SO THE COMPOSITION OF STARS CAN BE DETERMINED.

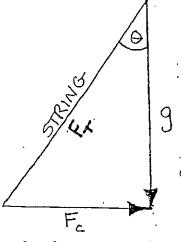
MOTION CAN BE DETECTED BY THE RED SHIFT OF THE SPECTRUM

Simple Instruments

The ASTROLABE, an ancient instrument that was used by sailors to find the angle of guide stars above the horizon, can also give some useful information. You can make one with an index card, a straw, a piece of string, and a metal washer. Tape the protractor shown below to your index card. Tape the straw along the top line. Punch a small hole where shown, knot one end of the string and put it through the hole. Tie your washer to the other end of the string and you have a simple astrolabe.



USING YOUR ASTROLABE: There are several interesting uses for an astrolabe. On rides that spin horizontally, like carousels and rotors, point your astrolabe toward the center of the circle and read the angle away from vertical that the weighted string makes. You can then use some simple trigonometry to find the centripetal force. It turns out that the tangent of the angle away from vertical (let's call it Θ) equals the ratio between the centripetal force and the force due to gravity. Here's how the math works out.



F_T is the resultant (total) force that the string must exert to resist gravity and keep the weight moving in a circle.

Fe is the centripetal force

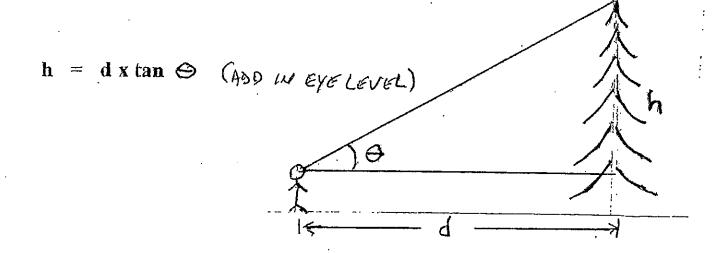
g is the force due to gravity

is the angle away from vertical as measured on the astrolabe

In a right triangle, the tangent of an angle is defined as the ratio of the opposite side to the adjacent side. In other words, the tangent of the angle away from vertical equals the centripetal force divided by the force due to gravity. This means that the centripetal force equals the tangent of the angle you measured in units of g-force, or:

$$F_c = \tan \Theta (g)$$

Another great use of an astrolabe is to find the height of something. Explorers like Lewis and Clark used an instrument like this to make measurements. If you know the distance to the base of an object, and the angle of elevation from that spot to the object, the height of the object is:



Explorers like Lewis and Clark used astrolabes to help them find the height of landmarks they encountered on their journeys. At an amusement park, an astrolabe can help you find the centripetal force for rides that circulate horizontally. You will use some simple trigonometry and the tangent table below.

TANGENT TABLE

Use this tangent table for the examples in this package.

Angle	Tangent	Angle	Tangent	Angle	Tangent '	Angle	Tangent	Angle	Tangent
0	0.0000	19	0.3443	38	0.7812	57	1.5396	76	4.0091
_1	0.0175	20	0.3639	39	0.8097	58.	1.6001	77	4.3295
_2	0.0349	21	0.3838	40	0.8390	59	1.6640	78	4.7023
_ 3	0.0524	22	0.4040	41	0.8692	60	1.7317	79	5.1418
4	0.0699	23	0.4244	42	0.9003	61	1.8037	80	5.6679
_5	0.0875	24	0.4452	43	0.9324	62	1.8804	81	6.3095
_6	0.1051	25	0.4663	44	0.9656	63	1.9622	82	7.1099
7	0.1228	26	0.4877	45	1.0000	64	2.0499	83	8.1372
_8	0.1405	27	0.5095	46	1.0354	65	2.1440	84	9.5045
9	0.1584	28	0.5317	47	1.0722	66	2.2455	85	11.4157
10	0.1763	29	0.5543	48	1.1105	67	2.3553	86	14.2780
11	0.1944	30	0.5773	49	1.1502	68	2.4745	87	19.0404
12	0.2125	31	0.6008	50	1.1916	69	2.6044	88	28.5437
13	0.2309	32	0.6248	51	1.2347	70	2.7467	89	56.9168
14	0.2493	33	0.6493	52	1.2798	71	2.9033		3013100
15	0.2679	34	0.5744	53	1.3269	72	3.0767		
16	0.2867	35	0.7001	54	1.3762	73	3,2698		
17	0.3057	36	0.7265	55	1.4279	74	3.4862		
18	0.3249	37	0.7535	56	1.4823	75	3.7306		1